Learning and Assessing Mathematics through Reading and Writing

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Students learn mathematics more effectively and more deeply when reading and writing is directed at learning mathematics. Although reading and writing in mathematics may necessitate more skills and practice to master, the mathematical learning derived from reading and writing mathematics far outweighs the burden it places on teachers and students. This paper explores the vital role of purposeful reading and writing in the mathematics classroom and outlines some techniques to promote successful integration of reading and writing in mathematics.

Calls for reading and writing across the curriculum have long been a part of educational reform movement. While many of these calls focus on increasing student literacy cart blanche, more specific calls for increasing reading and writing in mathematics curriculum are being grounded upon the understanding that students learn mathematics more effectively and more deeply when reading and writing is directed at learning mathematics. The National Council of the Teachers of Mathematics' Principles and Standards for School Mathematics (NCTM, 2000) states, "Students who have opportunities, encouragement, and support for speaking, writing, reading, and listening in mathematics classes reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically." (p. 60)

In the preliminary preparation for a series of research projects, the authors of this paper presented a number of workshops for K-12 preservice and inservice mathematics teachers. The purposes of these workshops were to provide mathematics educators with information regarding learning mathematics through reading and writing while simultaneously gleaning further understanding from their experiences, opinions, and practices. Anecdotal evidence offered by workshop participants was recorded. In this paper, comments or opinions cited as "(WP-S, 2006)" or "(WP-T, 2007)" are from anonymous students or teachers participating in these workshops in the respective year indicated. This anecdotal evidence has contributed greatly to the findings and activities outlined in this paper.

This paper explores the vital role of purposeful reading and writing in the mathematics classroom, the learning which results from such, and outlines some techniques to promote successful integration of reading and writing in mathematics.

Reading and Writing in the Mathematics Classroom

Beyond word problems and open-ended questions concluding exercise sets in the textbook, most students report that they do not read and write in mathematics classrooms and/or only read the smallest amount of text necessary to accomplish homework (WP-S, 2006). Others only investigate the examples or the diagrams provided in the text and rarely read any of the text between such (WP-S, 2007).

Research supports these students' assertions. Alvermann & Moore (1991) describe the common reading in the classroom as primarily textbook focused and teacher directed. Unfortunately, textbook focused and teacher controlled classroom reading leads to students perceiving themselves as passive receivers of knowledge (Siegel & Fonzi, 1995; Lemke, 1990) rather than interactive participants socially involved in negotiating understanding (Ernest, 1991; von Glasserfeld, 1991) through communities of practice (Resnick, 1988; Schoenfeld, 1992). While many teachers have students inspect diagrams in the text, few ever direct them to read paragraphs in the text (WP-T, 2007). Other teachers report that their own familiarity with the subject matter dissuades them from reading the classroom text (WP-T, 2007).

To fulfill school or district writing requirements, many mathematics teachers report that they annually assign writing biographical reports to their students (WP-T, 2006). Although Sipka (1990) discusses numerous types of formal and informal mathematical writing appropriate for the classroom (free writing, mathematics autobiography, writing letter to teachers, proof writing, and rewriting lecture notes), few of these are regularly observed in the classroom. Baroody and Ginsburg (1995) have discovered that most classroom mathematics writing has the goal of reproducing text in

the same style as found in the classroom textbook .

Informal journal writing (Borasi & Rose, 1989), informal creative writing (Alvermann & Moore, 1991), formal journal writing (Waywood, 1992; Clarke, Waywood, & Stephens, 1993) and expository writing (Venne, 1989) are the most common forms of writing seen in mathematics classrooms. Rothstein, Rothstein, and Lauber (2007) add a significant list of additional types of writing appropriate for the mathematics classroom. As different types of reading connote different purposes for such, different types of mathematical writing assignments also connote differing purposes for the experiences. Rose (1989) suggests that writing in mathematics benefit students as writers, the teachers as readers, and student-teacher interactions and Allen (1991) recommends that writing mathematics can be effectively used for a student to assess his or her own work.

Today, many state and national standardized tests require precise reading and complete written recordings of heuristics and solutions to many questions. Regrettably, many students are not regularly introduced to these experiences and many teachers are frustrated with standardized exam questions which are dissimilar to those posed in the textbooks and in the classroom (WP-T, 2006). Summarily, much of the reading and writing currently occurring in mathematics classrooms avoids the rigorous experiences which lead to meaningful mathematics learning.

The Skills of Reading and Writing in Mathematics

Mathematics texts are more conceptually dense than almost any other type of text (Brennan & Dunlap, 1985; Culyer, 1988; Thomas, 1988). The reading of mathematics often requires a process of reading text, inspecting associated diagrams, reading more text, considering some symbolic expressions, more text, more graphs, and etc. (Freitag, 1997; Noonan, 1990). Additionally, reading mathematics often differs from other types of reading because in addition to reading left to right and top to bottom, students must jump around the page to associate text with tables, graphs, symbols, and vice versa. Thus, the process is not linear and it cannot be assumed that a student who can read other types of text well can read mathematics with the same understanding. Nor can it be assumed that students who can read math texts fluently, actually understand or comprehend the text (Robb, 2003). Mathematics itself is fraught with linguistic and symbolic nuances which are difficult to navigate (Adams, 2003; Reehm & Long,

1996). Rheem and Long (1996) and Fuentes (1998) insist that reading mathematics is sufficiently different so to necessitate that students receive direct instruction in such.

Writing mathematics also requires different and additional skills than other typical scholastic writing assignments. Writing mathematics often requires a solid understanding of numeric, symbolic, graphical, and verbal representations, their uses, and their interconnections (Freitag, 1997). Since the language of mathematics is extremely precise and terse, writing mathematics requires specific instruction, an in-depth understanding of the vocabulary, and a great deal of practice. Students need to be trained to write mathematically (Shibli, 1992) and students who do not receive such training reap no positive benefits from the exercise (Moore, 1993). Generally, in typical classrooms, there are currently too little and too few types of mathematical writing experiences for students to become proficient mathematical writers (Marks & Mousley, 1990).

Learning through Reading and Writing in Mathematics

Bishop (1988) considers reading as a mode of learning. Theoretically grounded on Rosenblatt's transactional theory of reading (Rosenblatt, 1978, 1985, 1994), numerous researchers have come to the belief that students learn mathematics through reading mathematics (Borasi, Siegel, Fonzi, & Smith, 1998; Borasi & Brown, 1985; Borasi & Siegel, 1990; Siegel & Borasi, 1992; Siegel, Borasi, & Fonzi, 1998).

Writing is considered by Emig (1977) as a unique and potent form of learning. Writing in mathematics is a learning experience which deepens mathematical understanding (Rose, 1989) and extends students thinking and understanding (Shepard, 1993). Numerous studies report that students demonstrate greater mathematical understanding and learning through writing to learn mathematics (Brandau, 1990; Drake & Amspaugh, 1994; Doherty, 1996; Gopen & Smith, 1990; Grossman, Smith, & Miller, 1993; Miller, 1992; Nahrgang & Peterson, 1986; Pugalee, 1997; Rose, 1989, 1990; Stehney, 1990; Porter & Massingila, 2000). Guckin (1992) and Youngberg (1989/1990) demonstrate that college algebra students scored higher on some exams through writing to learn mathematics programs. Writing mathematics may lead to students increasing in content knowledge (Borasi & Rose, 1989) and recognizing and generating connections (Bradley, 1990).

However, Porter and Masingila (2000), Morgan (1998), and Powell and Lopez (1989) caution that little comparative research verifies the claims of writing to learn mathematics. Thus, one must carefully define "learning" in this context. Most of the researchers previously listed who indicate that students learn through writing mathematics do not necessarily mean that scores on assessments of content knowledge increase for students who are in more writing intensive mathematical experiences. Rather, since the making of connections among mathematical concepts is inherent within mathematical writing, most researchers equate the creation of, and focus on, these connections as tantamount to learning. Thus Porter and Masingila (2000) contend that writing to learn mathematics most benefits students by requiring them to investigate and consider mathematical concepts and connections and practice communicating such to others.

Reading, Writing, and Multiple Representations

In the Principles and Standards (NCTM, 2000), mathematical representations are generally recognized as symbolic (algebraic), verbal, graphical, and tabular (numeric). The term, multiple representations, connotes using more than one of these representations within a problem-solving situation. A growing body of research advocates the adoption of curricula which utilizes multiple representations. Some researchers emphasize the value in multiple representations which resides in the connections which are created (Knuth, 2000a; Romberg, Carpenter, & Fennema,1993). Each mathematical representation (verbal, tabular, graphical, or symbolic), carries particular strengths and weaknesses (Dugdale, 1993; Keller & Hirsch, 1998; Knuth, 2000a). Keller and Hirsch state that the connections inherent in multiple representations lead to increases in student conceptual understanding. Dugdale believes that when students are not constrained to one representation and are allowed to utilize multiple mathematical representations, problem-solving skills can be strengthened. When students lack the mathematical sophistication to select among many representations the one which they believe will most likely lead to the solution of a problem, some believe that students' problem-solving abilities are hindered (Keller & Hirsch, 1998; Knuth, 2000b; Lloyd & Wilson, 1998).

It is important to encourage students to represent their ideas in ways that make sense to them, even if their first representations are not conventional ones. It is also important that they learn conventional forms of representation to facilitate both their learning of mathematics and their communication with others about mathematical ideas. (NCTM, 2000, p. 67)

Since writing mathematically necessitates the use of verbal expressions, numbers, symbolic expressions, and graphical representations, mathematical writing can be simultaneously considered a single representation or the conjoining of all representations. As the latter, since conceptual connections are created through reading and writing in mathematics and simultaneously through the investigation and application of multiple representations (Knuth, 2000a; Romberg, Carpenter, & Fennema, 1993), it can be argued, as does Keller and Hirsch (1998), that these connections lead to increase in students' conceptual understanding.

The understanding and use of multiple representations are innately present in nearly every mathematics reading and writing task. Intrinsically, all reading and writing tasks—arguably the culmination of NCTM's communication standard—also necessitate reasoning skills which interweave mathematical connections among topics and representations. Altogether then, few learning experiences more completely encapsulate and exemplify the NCTM process standards and lead to the potential of learning mathematics.

Reading and Writing about Math versus Reading and Writing in Math

Although there have been numerous successful educational reform efforts centered on increasing reading and writing across the curriculum, most have insufficiently defined whether the purpose for such in mathematics is to increase student literacy or to increase mathematical knowledge. With this ambiguity in place, many have failed to distinguish reading and writing **about** mathematics from reading and writing **in** mathematics. Therefore, reading and writing biographies of famous mathematicians has upstaged reading and writing regarding the nature and uses of the discriminant of a quadratic polynomial. A plethora of entertaining literature now purports to develop mathematical interest among elementary, middle grades, and high school students (picture books, comic books, mystery novels). Unfortunately, most of these again fall into the camp of reading **about** mathematics rather than reading mathematics itself. Many educators are seduced into using these attractive materials without truly integrating reading and writing in the teaching and learning of mathematics. This is not to say that these resources should

not be used, rather to suggest that these books be used as a springboard for initiating deeper conversations about the underlying principles of mathematics.

While Borasi et al. (1998) advocate reading in many genres of mathematics related literature and more informal mathematics writing is supported by Borasi and Rose (1989) and Alvermann and Moore (1991), herein we equate these recommendations with reading and writing **about** mathematics. More consistent with the position taken herein are the types of formal journal writing and expository writing investigated by Waywood (1992), Clarke et al. (1993), and Venne (1989).

The Goal for Reading and Writing in the Mathematics Classroom

Biancarosa and Snow (2006) contend that reading is a central skill associated with life-long learning. NCTE and other professional organizations propound that literacy (reading, writing, and verbal communication) are all necessary skills in the development and continuation of life-long learners. In mathematics, no less so, reading and writing literacy in mathematics are essential for the life-long learning of mathematics. Therefore, mathematics educators must not falter from promoting that the goal for increasing reading and writing in mathematics classrooms must unequivocally be increasing mathematical learning. This understanding is instrumental for selecting appropriate classroom tasks and texts.

NCTM's communication standard states that students learn through collaboration, discourse, and reading and writing in mathematics (NCTM, 2000). Since standardized test questions are increasingly open-ended, requiring students to read, understand the question, and then compose responses, reading and writing are no longer seen as interesting add-ons to mathematics curricula; rather, they are recognized as instrumental tools to deepen student mathematical understanding. Unfortunately, all too few resources and techniques have been provided to teachers to employ in their classes (Marks & Mousley, 1990).

A Model for Reading and Writing in Mathematics

A number of models for reading and writing exist for educators to consider. The Standards for the English Language Arts (NCTE/IRA, 1996) describes a model in which Context leads to define Purpose, Content, and Development, and altogether affect the Learner. A simpler model promoted by the North Carolina Department of Public Instruction's (NCDPI) Standard Course

of Study for English Language Arts (NCDPI, 2004) demonstrates that reading and writing are constructed on the foundation of Purpose, Audience, and Context. Barton and Heidema (2002) have created a model which connects reading with mathematics and builds upon the threefold foundation of the Reader, Climate, and Text Features.

Each of these models represents its own respective strengths and weaknesses. The Standards for the English Language Arts (NCTE/IRA, 1996) appropriately distinguishes context from content, considers the development of ideas in the text (reading or writing), and recognizes the learner as NCELA's audience. Barton and Heidema equate text features with NCTE's dimension denoted as development. As in other models, albeit not explicitly stated, Barton and Heidema consider the importance of creating a proper classroom climate for supporting and promoting reading in the classroom. Unfortunately, their model becomes overly complex for many educators to effectively apply in mathematics classes. The initial three dimensions of Reader, Climate, and Text Features become further developed into secondary and tertiary dimensions of Vocabulary, Text Style, Audience Appropriateness, Test Coherence, Text Structure, Text Organization, and Text Presentation. Thus, altogether, their model becomes convoluted by the introduction of numerous categories.

This article unites these and a number of additional models to create a model for reading and writing which is more useable by mathematics educators (See Figure 1.). Herein, Purpose (the reason or goal for the reading or writing), Context (the situation), Audience (the readers of the text), and Climate (the classroom Zeitgeist encouraging or hindering students' willingness and interest in participating in reading and writing in the
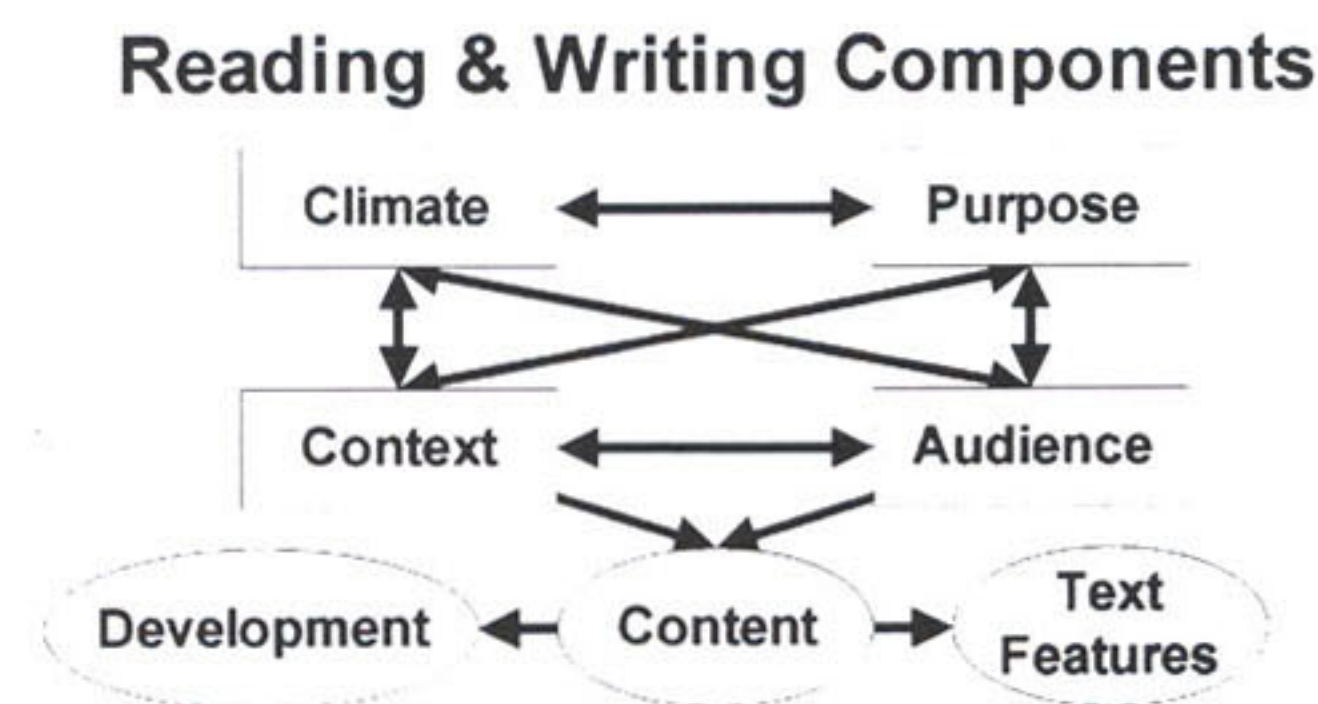
Reading & Writing Components

Figure 1. Reading and writing components.

classroom) all work in tandem. If any component is missing, students are less apt to willingly participate in reading and writing assignments. For instance, if students are unclear regarding the purpose of a text, motivating them to read the text is difficult. If the audience for which a text is to be written is ambiguous, it is difficult for students to appropriately write about issues. In respect to writing (Burton & Morgan, 2000) and reading (Borasi et al. 1998), students' gaining of mathematical understanding from these experiences is correlated with their understanding of the purpose of the writing assignment or the author's purpose for the text being read.

In this model, the selection of mathematical content for any reading or writing assignment is most directly a function of the audience and the context for the assignment. For any audience, context, and content, the development (organization, sequence, and flow) of ideas within the content and the text features (vocabulary, sentence structure, paragraph and section length, accompanying diagrams and examples, and number and type of practice exercises) must be commensurate with the readers' understanding and ability to learn.

Demonstrating the model

In workshops for mathematics teachers, the authors use an activity to further elucidate the model. Teachers are asked to work in pairs and "write a definition for the mathematical term factor." When workshop participants complete the task, they are asked leading questions such as: What did you write for a definition?; Was factor a noun or an adjective?; Did you take into account the audience for whom the definition was intended?; and would/should the definition be different for middle grades students, secondary students, students in number theory, or students in algebra?. Through this activity, teachers are made aware of the importance of considering the purpose, audience, and context for a simple example. The activity also highlights the need for a common understanding of terms. Though "factor" is a word with which all participants are familiar, the context (the situation) dictates the definition and use.

Workshop participants are then asked to discuss whether a more favorable climate supporting such a task could have been created prior to spontaneously asking them to extemporaneously develop a definition. They immediately come to realize that as proper motivation affect them, their students are also indisputably affected by the climate teachers create in their own classrooms. The workshop sessions demonstrate to mathematics teachers the significance of the interplay among purpose, audience, context, and climate in the model.

Improving the Classroom Climate for Reading and Writing in Mathematics

As denoted in the model, addressing purpose, audience, and climate are necessary for effective learning of mathematics through reading and writing. Often, the climate for learning through these modalities is immediately enhanced or sullied by the level to which students understand the purpose of the experience and the degree to which the experience was appropriately geared to the student audience. Numerous teachers share that increasing reading and writing in the mathematics classroom is virtually impossible unless the groundwork for a positive classroom climate has been established (WP-T, 2006). Among those teachers who have been most successful in increasing reading and writing in the learning of mathematics, two recommendations have been provided most often.

Expectations

Reading the textbook and other textual materials must become a recognized and regular expectation in the class (WP-T, 2007). Both in classroom activities and homework assignments, the necessity of reading mathematical texts must be emphasized. Writing mathematics must also be correlated with clearly defined expectations. Reading and writing must be recognized by teachers and students as a natural and expected means through which mathematics is learned.

Modeling

Teachers who have found success in integrating literacy in mathematics teaching and learning predominantly indicate that student success is born initially from teachers modeling the importance of reading and writing in the mathematics classroom (Aiken, 1977; Blanton, 1991) and that these are desirable skills and techniques through which to learn. Teachers reported that students were more ready to consider, adopt, and utilize reading and writing strategies in mathematics after they witnessed teachers utilizing reading and writing techniques in their lessons and observed teachers discussing mathematical materials that they had read or written in order to themselves learn mathematics (WP-T, 2007). Student interest in reading and writing mathematics was further promoted through assignments that develop mathematics content (Thompson, Austin, & Beckmann, 1999). It is, therefore, imperative that mathematics teachers assume the responsibility for

teaching the necessary reading skills in mathematics (Reehm & Long, 1996).

Avoiding Negative Consequences in the Classroom

Since students experience and witness little focus on reading and writing in traditional mathematics instruction, they often perceive such tasks as laborious and dislocated from mathematical studies. Acknowledging that students typically balk against any changes in their routines, it is important that mathematics teachers introduce these novel reading and writing assignments in ways which will not automatically engender even more antipathy (WP-T, 2006). The following recommendations are intended to ensure that the inclusion of reading and writing in the mathematics classroom is accepted positively.

Set the purpose for reading and writing

When students are directed to read a passage or write a response and are not provided adequate motivation for such, frustration may arise. It is incumbent for mathematics teachers to prepare students for these tasks by clearly delineating their respective purposes (WP-T, 2006). Students must come to value these assignments and their inherent roles in the learning of mathematics. The purpose for the assignment must be clearly understood (Burton & Morgan, 2000; Borasi et al., 1998). Although this may take some time commitment on the part of the teacher, the results will be worthwhile.

Appropriate assignments

Since literacy is gained incrementally and mathematical reading and writing assignments are novel experiences to most students, it is important for teachers to carefully plan initial assignments and not make them overly complex or difficult to successfully complete (WP-T, 2007). Assignments must be grade and age appropriate (Armbruster, 1996; Readence, Bean, & Baldwin, 2001), be selected in respect to deliberately accomplishing specified mathematical learning goals, and be commensurate with students' prior knowledge (relevant vocabulary, concepts and connecting ideas) (WP-T, 2006).

Techniques for Increasing Reading and Writing in Mathematics

The recommendations for increasing reading and writing in mathematics provided herein are separated into different types: recommendations which are general to increasing reading and writing in any educational setting or subject; examples for mathematical studies which are generalizable to other topics and

questions; and mathematical teaching methodologies. The latter is reserved for its own section.

While a number of the recommendations in the following discussions are directly generated by a natural application of the newly configured model provided previously, some of the earlier and more generalizable recommendations evolve from the application of the models which were employed in the development of the new model. Therefore, whether employing the model presented herein or other models, it is important for teachers and curriculum developers to continually use an effective, workable model as they plan instruction.

General Recommendations

Print rich environment. All classrooms should be print rich environments, balancing textual quality with quantity (Robb, 2003). In mathematics classes, these texts should include textbooks, dictionaries, encyclopedias, educational journals, maps, science texts, biographies of mathematicians, mathematical posters, and various books. Since students are more interested in learning when they clearly recognize applicability to their own lives, teachers can maintain an area of *mathematical connections* in the classroom, containing mathematical publications connected to students' interests and current events. The internet can also provide an endless source of reading materials for mathematics students in nearly all grades. Print materials should also include samples of student and teacher writings on mathematical topics. Teachers can work with their school's media specialists and librarians to create a monthly focus on a particular topic.

Highlighting text structures and vocabulary. Research supports a strong correlation between students' comprehension and their knowledge of text structures (Robb, 2003, Fielding & Pearson, 1994) and between students' mathematical comprehension and their knowledge and use of multiple representations (Piez & Voxman, 1997; Romberg et al., 1993). Since narrative text differs from expository text, knowledge of narrative text structure is helpful when completing expository tasks like reading or writing mathematics. Teachers should offer instruction in understanding the text structures and vocabulary associated with respective course material (Robb, 2003). For example, mathematics teachers should demonstrate how the use of vocabulary, text structure, ideational sequencing, and etc., all work in tandem to create a text appropriate for the understanding of mathematics.

Note taking. Requiring students to practice clear,

precise, and organized note taking during instruction and reading and jot down questions that may arise may be the most natural application of reading and writing in the mathematics classroom and should be encouraged.

Mathematical Recommendations

Many successful teachers provide specific techniques to increase reading comprehension in mathematics classrooms (Barton & Heidema, 2002; Blanton 1991). Most of these techniques can be modified to be grade appropriate by teachers. Some of these recommendations are directed at ameliorating some of the negative reading and writing habits more frequently observed in mathematics classes.

Reorganizing the Text and Exercises

The natural layout of mathematics textbooks – numerous pages of discussion, examples, more discussion, and more examples all followed by a few pages of compacted homework exercises – may be an inherent inhibitor to students reading the text. Students typically skip all the explanatory text and focus attention directly on exercises, only returning to the text when sufficiently stymied by a problem (WP-T, 2006, 2007; WP-S, 2006, 2007). Teachers can have students read a textbook selection and immediately attempt the respective exercises. This reordering of text and exercises helps acculturate students to the notion that the text is valuably instructive and not simply meant to be a backup when stumped during homework (Pinne, 1983). This technique for secondary and college students is analogous to the techniques discussed by Borasi et al. (1998) in respect to "Cloning an Author" and "Using Cards".

Blue Box Techniques. Mathematics textbooks typically share common publishing traits. Important information, such as definitions and theorems, are placed in colored text boxes (all too often blue). An effective technique to ensure that students read and digest the information in the blue boxes is, after they have claimed to have read and understood the encapsulated information, to have them close the book and create a problem scenario or example which would necessitate or demonstrate the use of the information in the text box. This technique melds the recommendations and findings of in respect to formal mathematical writing (Clarke et al., 1993; Waywood, 1992) and expository writing (Venne, 1989) with more creative writing styles (Alvermann & Moore, 1991; Borasi & Rose, 1989; Sipka, 1990).

Initially, students will find this methodology quite challenging, until they realize that a thorough reading of the information often leads to an understanding which can be applied to various scenarios. Misunderstanding the reading occasionally leads to students creating examples which either do not work correctly or are overly complex to be solvable. In each case, these misunderstandings lead directly to teachable moments and rich classroom discussions. Experiencing this learning technique, students report gaining far deeper understanding of the mathematics than they had previously experienced under traditional instructional techniques.

Changing Definitions and Theorems. Some mathematics educators have suggested as classroom experiences that teachers purposefully and carefully alter words in definitions and theorems (WP-T, 2006). After the teacher has changed a number of words in the text, students openly discuss the validity of the modified statement. Discussions lead to both more careful reading and an appreciation for the precision by which mathematics is written. Numerous mathematics educators and researchers advocate that students' understanding of mathematical vocabulary is significantly connected to their understanding of mathematics (Earp & Tanner, 1980; Helwig, Rozek-Tedesco, Tindal, Heath, & Almond, 1999; Stahl & Fairbanks, 1986). This experience of teachers altering definitions/theorems and students being assigned to debate or correct erroneous statements leads students toward the mastery of the vocabulary and concepts.

Comparing Texts. Today, numerous mathematical discussions on any particular topic are readily accessible through textbooks and the internet. Students can compare and contrast these various texts and discuss the techniques which authors use to communicate ideas (flow of concepts, use of diagrams and graphics, use of vocabulary, and etc.). This develops both their ability to read mathematics and helps them begin to understand how to effectively write mathematically. Moreover, when students compare different texts on the same concept, they become more familiar with the authors' conceptualizations of purpose, audience, and context as proposed in the previous model.

Unpacking mathematical texts. Mathematics texts are significantly different from texts in other fields. The terse nature of the language of mathematics, together with the need for extensive verbal explanation of compressed symbolization and numerous diagrams, tables and graphs, creates a dichotomy which student readers

often feel difficult to navigate. Reading mathematics challenges students to acquire mathematical comprehension through reading simultaneously numerals, symbols, and words (Adams, 2003). Unpacking these texts is necessary for students to gain understanding in the mathematics (Armbruster, 1996).

Applying Findings to the Classroom Instruction

The following examples are provided to demonstrate some of the many techniques which may be used in mathematics classroom to increase mathematical learning through reading and writing. These more specific techniques are intended to be interwoven with the more general examples provided above.

In many of the techniques presented below, students gain understanding and experience in working with the model above. For a given mathematical topic, students must determine the *purpose, audience,* and *context* for their responses and decide which manner of *development* and what *text features* are most appropriate for the *audience,* meet the purpose of the response, and develop the *climate* which they may wish to generate.
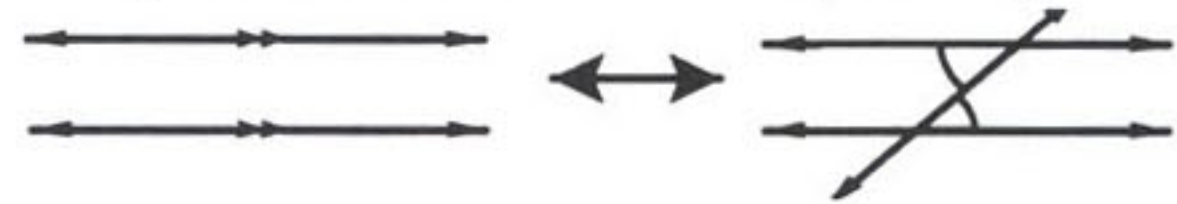
Integrating Multiple Representations

Borasi et al. (1998) have promoted some effective reading strategies to be used in the learning of mathematics. Among these is the Sketch-to-Stretch activity wherein students are to draw nonlinguistic representations of what they understand from the text and engage in elaborating on their own thoughts. "When students elaborate on knowledge, they not only understand it in greater depth, but they recall it much more easily" (Maranzo, Pickering, & Pollock, 2001, p. 74). Through this experience and the subsequent communication of their learning to other students, students reported that they better understood the mathematics being investigated.

Many types of charts and diagrams fall under the mathematical representation commonly denoted as graphs. Thus, research in graphical organizers can shed more light on techniques appropriate for the application of reading and writing in the classroom. Numerous scholars have noted that students' use of graphic organizers (both reading and creating) have significantly improved student understanding of concepts (Jones, Palincsar, Ogle, & Carr, 1987; Monroe, 1997; Monroe & Pendergrass, 1997) and contend that through these representations students are often able to discover and restructure misconceptions (Fuentes, 1998). Readence et al. (2001) provide another version of this technique, stating that greater learning results

when students connect verbal concepts with pictures and diagrams.

Altogether, therefore, it is valuable for students to be able to translate concepts between representations in their learning of mathematics (Dugdale, 1993; Keller & Hirsch, 1998; Knuth, 2000b; Lloyd & Wilson, 1998). The examples in this section exemplify connections between reading, writing and multiple representations.

Example 1. Write the theorem depicted by this diagram.

Example 2. Draw a pictorial representation of the theorem: Given a line and a plane containing it, the points of the plane that do not lie on the line form two sets such that (1) Each of the sets is convex and (2) If P is in one set and Q is in the other, then segment PQ intersects the line.

Example 3. Explain one of the following theorems using both text and diagrams. "If tangent and secant lines are drawn to a circle from the same point in the exterior of the circle, the length of the tangent segment is the mean proportional between the length of the external secant segment and the length of the secant."

Example 4. Use text and pictures to explain the concept, "If $(x - c)^k$ is a factor of a polynomial $f(x)$, but $(x - c)^{k-1}$ is not a factor, then c is a zero of multiplicity k."

Example 5. Using both text and diagrams, tell all you can about the function:

$P(x) = a(x - b)^z(x + c)^y(x - d)^w(x + e)^v(x - f)^u(x + g)^t(x - h)^s$, where $a, b, c, d \in R$, $e, f, g, h \in C$, and $s, t, u, v, w, y, z \in N$.

Creating Your Own

Using recommendations from researchers and curriculum developers (e.g., Alvermann & Moore, 1991; Rothstein, Rothstein, & Lauber, 2007; Barton & Heidema, 2002) the following examples allows a student to both investigate a mathematical concept and express it creatively, thereby deepening his/her understanding of the concept.

Example 1. Create a real world problem/scenario for which the use of the following theorem would be part of the solution. "A point is on the perpendicular bisector of a line segment if and only if it is equidistant from the endpoints of the segment."

Vocabulary Building

Since the language of mathematics is simultaneously

verbal and symbolic, it is important to build mathematical vocabulary in both representations. Some have found strong correction between student knowledge of mathematical vocabulary and ability to solve problems (Earp & Tanner, 1980; Helwig et al., 1999; Stahl & Fairbanks, 1986). The following simple examples demonstrate the necessity of being fluent in both mathematical words and symbols.

Example 1. Fill in the blanks.
When $P(x) \div (x-c) = Q(x) + (R/(x-c))$ and $R = 0$,
$(x - c)$ _____ $P(x)$;
c _____ $P(x)$; and
$x = c$ _____ $P(x) = 0$.

Example 2. Fill in each blank with a symbol which would make the property of the real numbers true.

"_____ a _____ R, a _____ 0, _____ (a -1) _____ R, such that $a \times (a -1) = 1$"

Analyze and Explain

The non-linear processes of reading and writing mathematics require reading text, inspecting associated diagrams (graphs, tables, charts), considering symbolic expressions, and fluidly moving in and between each of these representations (Adams, 2003; Freitag, 1997; Noonan, 1990). Therefore, learning activities should begin at different representations and move toward others. To always begin at one representation and lead to others diminishes the students' understanding of the inherent strengths and weaknesses of each representation (Dugdale, 1993; Keller & Hirsch, 1998; Knuth, 2000b; Lloyd & Wilson, 1998).

Formal writing of mathematics, in addition to necessitating some level of command of multiple representations, also requires the understanding and skill of a highly refined and terse grammatical syntax. Thus, formal writing is only mastered through many experiences. The following examples both interconnect many of the previously stated findings among reading, writing, and mathematics and begin the assignments by focusing on different representations and leading to others.

Example 1. Explain what this diagram demonstrates.
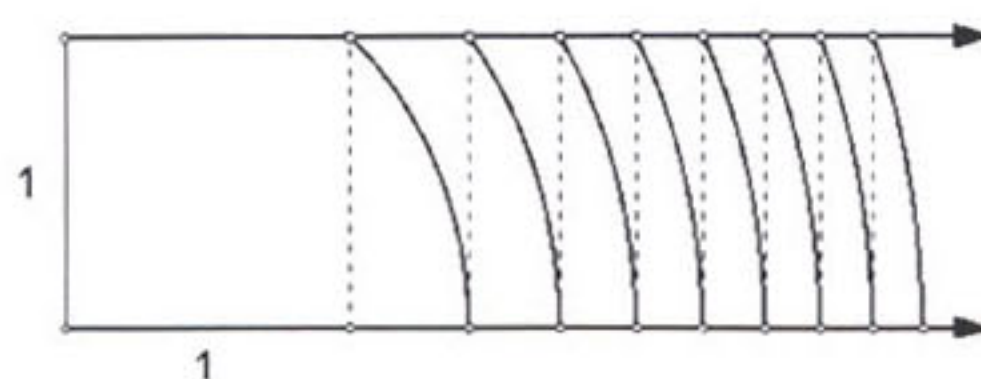
Example 2. Tell all you can about the function which would produce the accompanying truncated graph of $f(x)$.

Example 3. For each of the following rules for real numbers, state the conditions which would be required to ensure the statement is always true.

$$\sqrt[p]{\frac{\sqrt[m]{a}}{\sqrt[n]{b}}} = \frac{\sqrt[mp]{a}}{\sqrt[np]{b}}$$

Example 4. The following mathematical argument possibly requires the insertion of a number of intermediate steps in order for most students to fully understand the logic. Determine what those missing steps may be and where they should be inserted.

"Let $a = cr$ and $b = cs$. Then,

$$\sqrt[m]{a} \cdot \sqrt[n]{b} = \sqrt[m]{c^{\log_c a}} \cdot \sqrt[n]{c^{\log_c b}} = c^{\frac{n\log_c a + m\log_c b}{mn}} = \sqrt[mn]{c^{\log_c a^n + \log_c b^m}} =$$

$$= \sqrt[mn]{e^{\ln a^n + \ln b^m}} \ ."$$

Example 5. Explain the following.
"Let $x = a^m \times c^n \times d^p$ and $y = a^l \times b^r \times c^s$, where all variables are natural numbers. Then,

$GCF(x, y) = a^{\min(m, q)} b^{\min(0, r)} \times c^{\min(n, s)} \times d^{\min(p, 0)}$ and $LCM(x, y) = a^{\max(m, q)} \times b^{\max(0, r)} \times c^{\max(n, s)} \times d^{\max(p, 0)}$.

This leads to $GCF(x, y) = a^{\min(m, q)} \times c^{\min(n, s)}$ and $LCM(x, y) = a^{\max(m, q)} \times b^r \times c^{\max(n, s)} \times d^p$."

Conclusion

Reading and writing **about** math is not the same as reading and writing **in** math. Through introducing reading and writing mathematics as learning assignments, students have much to gain. Students who have opportunities, encouragement, and support for purposeful writing and reading in mathematics classes will have a more concrete grasp of concepts and be able to apply learning to alternate situations. There are multiple techniques to promote successful integration of reading and writing in mathematics and as mathematics teachers, we should explore these and share our successes. It is hoped that the discussions and examples provided here will be the springboard for continued exploration of more techniques through which to instruct mathematics.

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